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On instabilities arising during sedimentation of two-component mixtures of solids

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Experimental evidence is presented which shows that both uniform and differential sedimentation are unstable flows for relatively concentrated fluid suspensions containing particles of two different densities or sizes. The instability takes the form of a new type of fingering phenomenon which develops as a result of the two kinds of particles segregating laterally from each other.

1. Introduction

Most studies of sedimentation in solid–fluid systems have been confined to particles of *monodisperse* (uniform) size and density so that settling takes place with the particles distributed throughout the suspension in a spatially uniform manner. On the other hand, particles of varying size or density often sediment differentially in the vertical direction. Theoretical analyses of differential sedimentation and the interpretation of experimental data have invariably been based on the assumption that the spatial distribution of particles in any *horizontal* plane is uniform. However, we have recently discovered that dispersion in particle size or density can lead to far more complex types of behaviour. The purpose of this note is to report some experimental observations which suggest that, at least for bidisperse systems with sufficiently large differences in density or size, any form of spatial uniformity is unstable. The instability often takes the form of a density-driven fingering phenomenon.

In an attempt to learn more about the interaction between sedimenting particles, Whitmore (1955) replaced part of the continuum fluid with particles of equivalent (neutral) density; his appear to be the first observations on the settling of bi-disperse suspensions. At low total-solids concentrations he found only a slight retardation in the settling velocity, and he obtained values of the Einstein viscosity constant in reasonable agreement with theory. At high-enough solids concentrations, however, he made the remarkable observation that vertical streams developed in the suspension and these resulted in appreciable increases in sedimentation rates. Although the results of a number of subsequent studies dealing with sedimentation of bi-disperse suspensions have appeared in the literature (Richardson & Meikle 1961; Smith 1965, 1966, 1967; Phillips & Smith 1971; Davies 1968; Barnea & Mizrahi 1973; Lockett & Al-Habbooby 1972, 1973, 1974; Lockett & Bassoon 1979; Mirza & Richardson 1979; Davis, Herbolzheimer & Acrivos 1982; Greenspan & Ungarish 1982), the streaming phenomenon has gone largely ignored until only recently. In all of these studies it has been assumed that spatial uniformity prevails in any horizontal plane, although Phillips & Smith (1971), being aware of Whitmore's work, have alluded to the possible tendency for some sort of order to develop during differential sedimentation. That concentrated suspensions contaning particles of widely different sizes or densities may

	Buoyant–heavy system			Heavy–heavy system I			Heavy–heavy system II		
Property	Ceramic	PVC	Fluid	PVC	Glass	Fluid	PVC	PVC	Fluid
ρ (g/cm^3)	0.595	1.406	1.124	1.406	2.463	1.062	1.406	1.406	1.124
d (μm)	137	137	—	90	90	—	138	218	—
μ (cP)	—	—	4.949	—	—	1.80	—	—	4.949
C (vol. %)	16.5	17.5	66.0	17.5	17.5	65.0	9.5	14.3	76.2

TABLE 1. Physical properties of components in fluid–solid mixtures

undergo lateral segregation of the different species has been suggested by Davis *et al.* (1982), but no supporting evidence was given and the possible existence of local nonuniformities was considered no farther.

Whitmore's discovery that the presence of neutrally buoyant particles could result in convection was first taken up by Weiland & McPherson (1979). Upon adding *positively* buoyant particles to an otherwise uniformly settling suspension, they observed a rapid lateral segregation of the two species of particles into vertically directed streams. Enormous increases in settling rates resulted. Although a small amount of settling rate data has been reported since by Fessas & Weiland (1981, 1982), the phenomenon of fingering in systems of heavy and buoyant particles does not seem well known, and the possibility of the evolution of similar flows in sedimenting suspensions containing heavy particles exclusively (but of different densities or sizes) has never been examined.

2. Experimental

The particle–fluid systems used are listed in table 1. Visualization of the flow was achieved by coating one of the two types of particles with a fluorescent dye and photographing the flow under long-wave ultraviolet illumination in a darkened room. In this way, the coated particles were clearly distinguished from the uncoated ones because the former emitted brilliant light in the visible region while the latter virtually disappeared from view. Suspensions were contained between two clear Plexiglas plates having a spacing of 6 mm; these were 225 mm wide and 350 mm high for the buoyant–heavy suspensions, while for suspensions of only heavy particles the dimensions were 120 mm by 180 mm. The cells contained a large air bubble and they were tumbled end over end to achieve mixing. Particle agglomeration was prevented by using Triton X-100.

3. Observations and discussion

A sequence of photographs showing the evolution of fingering in the buoyant–heavy particle suspension is shown in figure 1. It can be seen that after the elapse of only 3 s (top left corner in the photograph) large-scale non-uniformities had already appeared within the suspension. By the 10 s mark (next frame) a fingering structure had already evolved, and it was well established within 20 s after the cessation of mixing. The streaming flow persisted as long as the heavy and buoyant particles were present together, but sedimentation returned to normal once vertical separation of the two types of particles was complete. Close examination of the buoyant phase (dark

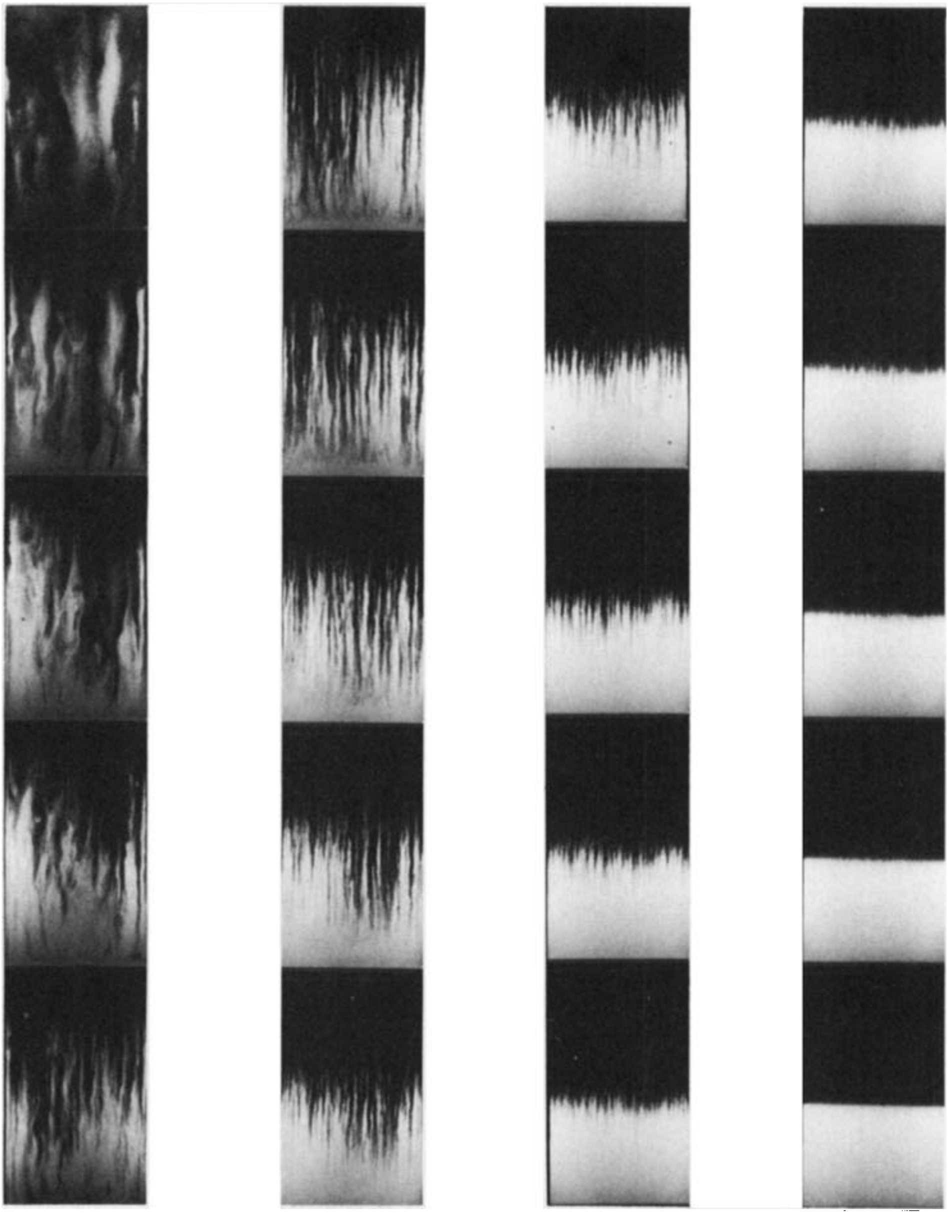

FIGURE 1. Sequence of photographs showing the evolution of fingering in the buoyant–heavy particle system. The first frame (top left corner) was taken 3 s after mixing was completed, the second frame 10 s after completion of mixing, and subsequent frames every 10 s thereafter. Frame width is 225 mm and heavy particles are dyed.

area in the photographs) that collected at the top of the sedimentation vessel failed to reveal the presence of heavy (dyed) particles. This indicates that the lateral segregation of the buoyant from heavy particles which takes place immediately after mixing stops results in fingers of suspension containing one or the other type of particle exclusively. The lateral separation is very clean indeed. Evidently, the buoyant particles have a strongly destabilizing effect on the sedimentation of the heavy particles, and *vice versa*. Readily visible non-uniformities appear with great

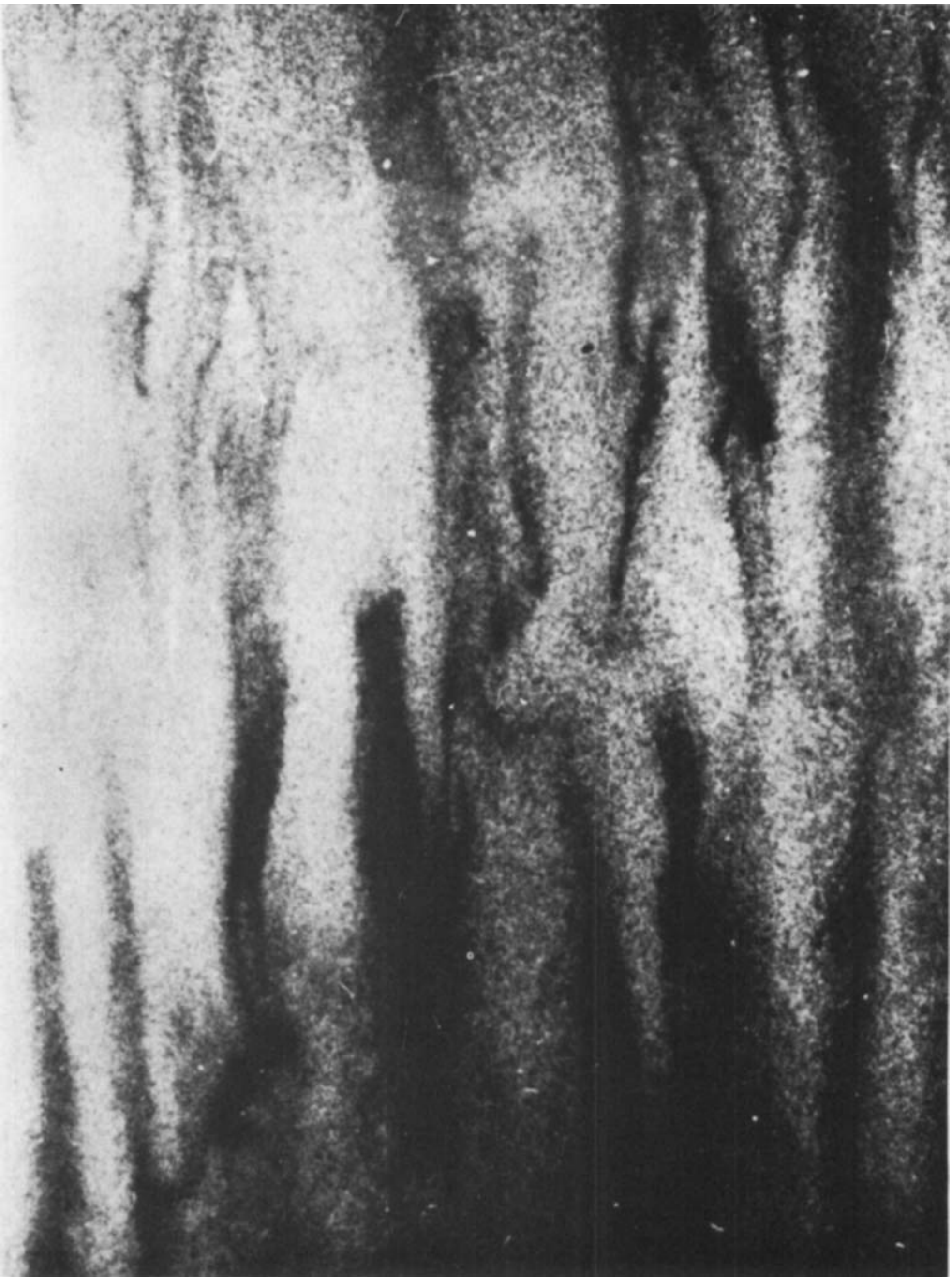

FIGURE 2. Photograph of heavy–heavy system I (equisize particles of different densities) taken 20 s after cessation of mixing. Frame width is 115 mm and PVC particles (less-dense phase) are dyed.

rapidity, and sedimentation returns to normality only when the two types of particles find themselves present alone once again.

This convective flow is reminiscent of salt fingers in double-diffusive convection but in this case, because of the relatively large particle size (about 100 μm), Brownian motion is insignificant and diffusion cannot play a role. Rather, the instability appears to be of Rayleigh–Taylor type, yet there are no density inversions to initiate

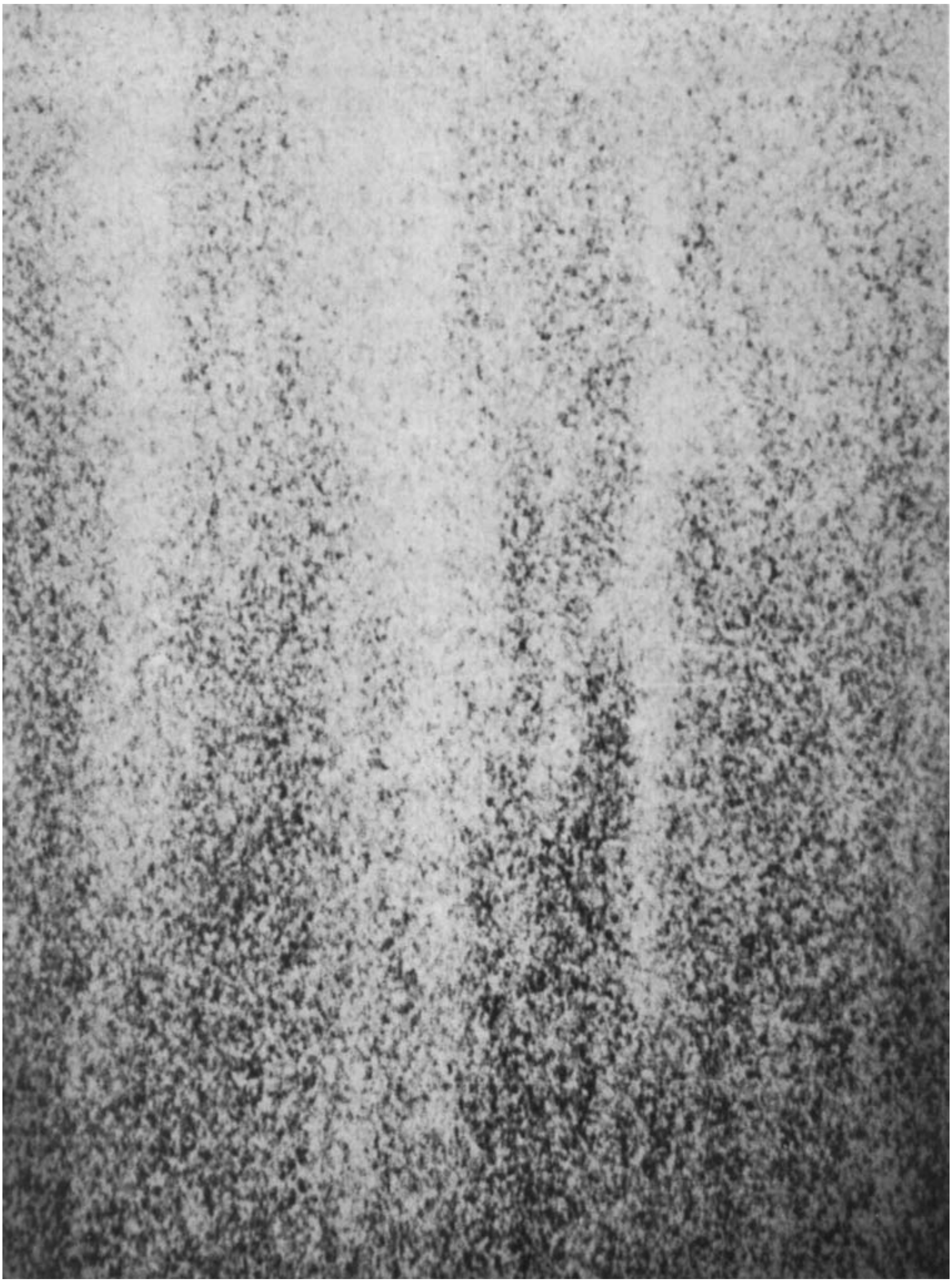

FIGURE 3. Photograph of heavy–heavy system II (equidensity particles of different sizes) taken 50 s after cessation of mixing. Frame width is 115 mm and smallest particles are dyed.

the flow since the particles were initially well mixed. However, there is relative motion between the two types of particles, and, as shown by Fessas & Weiland (1984), the greater this is, the higher is the convective velocity once the streams are established. It is the *relative motion between particles* that seems to cause the instability. It should be noted that if the total suspension concentration is too low (less than about 10% by volume) fingering is not observed; instead, both solid components settle at reduced rates. Thus a strong interaction between particles seems to be an essential ingredient in the formation of fingers.

Another interesting observation is the apparently regular spacing between fingers. A spatially periodic structure emerges from the initially uniform mixture. One might suppose that the period reflects the wavelength of the fastest-growing disturbance to the uniform basic state. However, the initially large-scale structure slowly evolves into one of increasingly finer scale, reaching an apparently steady periodicity only after the passage of several minutes. The continual splitting of large fingers into smaller ones suggests that the ultimate frequency of the periodic structure has more to do with the stability of the fingers themselves, wide fingers being unstable and so splitting into finer, presumably more stable ones.

On the basis of experimental evidence, we have suggested elsewhere (Fessas & Weiland 1984) that the flow field within fingers is uniform (shear-free) and that viscous drag caused by the relative motion of the fingers through their surroundings is confined to a thin shear or lubrication layer at the interface between fingers of one particle type and those of another. In this lubrication layer shear rates will be quite high. It can be seen in figure 1 that the large streams have wavy surfaces, perhaps indicative of an unstable flow. It is suggested here that the instability of the interface is driven by high shear rates in the lubrication film. As the streams become smaller in diameter, their velocity decreases until the point is reached at which the shear rate is too low to result in further instability with subsequent bifurcation into finer structure. There appears to be a critical shear rate below which the fingers are stable. Thus it would seem that the uniform mixture is unstable to long waves but that the ultimate structure of fine-scale results from nonlinear interactions within and between the convective flows themselves.

Although structure develops most rapidly and sharply .in systems containing buoyant and heavy particles, fingering can also result when all the particles are more dense than the fluid but are of two kinds, distinguishable by either density or size. Figure 2 shows fingering in a suspension containing particles of roughly the same size but of two different densities, both greater than the fluid. This photograph was taken 20 s after mixing was completed and the cell was set to rest. The structure is not as clearly developed as for the buoyant–heavy system after the same elapsed time, but is quite distinct nevertheless, and it evolved into a spatially periodic fingering structure after about 80 s.

Finally, a photograph of the fingering in a system containing particles of equal density but of two different sizes is shown in figure 3. The large particles, dark in the photograph, were about 220 μm in diameter so the photograph appears quite grainy. It can be seen that after about 50 s the streams are still fairly wide and relatively indistinct, but that lateral segregation into convective fingers has occurred and a structure has certainly developed.

It seems that in any system in which the solids concentration is high enough and the two types of particles can be distinguished from each other on the basis of size or density, spatially uniform sedimentation is unstable. At high total-solids concentrations the instability takes the form of convective streams or fingers which develop into a spatially periodic structure akin to salt fingers or Bénard convection cells. The cause of the instability, however, appears different from those responsible for salt fingers, Bénard convection or Rayleigh–Taylor instabilities.

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